OPEN SYSTEMS FORMULATION AND GAINING INSIGHT USING LAGRANGIAN BOND GRAPHS

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ABSTRACT

Models of a small-scale water rocket are developed as an example of open system modeling by both the bond graph approach and a more classical method. One goal of the development is to determine the benefits of the bond graph approach into affording insight into the system dynamics. Both modeling approaches yield equivalent differential equations as they should, while the bond graph approach yields significantly more insight into the system dynamics. If a modeling goal is to simply find the system equations and predict behavior, the classical approach may be more expeditious. If insight and ease of model modification are desired, the bond graph technique is probably the better choice. But then you have to learn it!

NOMENCLATURE

$A(x)$	cross-sectional area of differential mass
A_L	cross-sectional area at internal air-water interface
A_0	cross-sectional area at nozzle
c_d	drag coefficient
dm	differential mass of fluid element
F_d	drag force
g	gravitational acceleration
-in	subscript denoting "initial"
I	hydraulic inertia
L	height of fluid column in rocket
m_r	rocket mass
m_t	total mass
p	linear momentum
P_a	pressure of air in rocket
P_{atm}	atmospheric pressure
Π_e	power flow to environment
Q	fluid volumetric flowrate
T_p	kinetic energy
T_f	kinetic co-energy
v	rocket velocity, positive up
V	volume of water in rocket
V_a	volume of air in rocket
ς_q	potential energy
x	height of differential mass from fluid exit plane
Γ	pressure momentum
ρ	fluid density

INTRODUCTION

It is sometimes said that the "best" engineering model is the simplest model that answers the questions at hand. This philosophy is true if you know the questions before you build the model and, most of the time, you probably do. What about situations where you don't know all the questions to ask? What if you need more insight into a situation before you can pose better and more specific questions? This is where you need a model that gives a sufficient amount of insight into the physical situation. In the realm of engineering systems, this means insight into the relationships of the important variables: power, energy, momentum, motion, etc., insight into not just how individual variables behave, but into the interactions and relationships among key variables.

A hypothesis of this paper is that bond graphs afford a dynamic system designer or analyst an excellent tool for understanding the dynamics of open systems, systems that exchange mass and energy convectively with the environment. The understanding afforded is perhaps better (or more easily attained) than that possible by classical modeling methods or by using "off-the-shelf" modeling software. This is not to knock these other approaches. They are often powerful and can answer a majority of the engineering questions posed in straightforward ways.

To this end, a dynamic model of a water rocket is presented as an example of an interesting open system using Lagrangian bond graphs. The initial question is: "How does one maximize the vertical flight of the rocket?" The Lagrangian approach forces one to examine energy and momentum behavior in detail and perhaps find relationships that would otherwise remain hidden. As the model is constructed, insight is revealed. In comparison, a rocket model is also developed classically and the approaches are compared, both in the

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model construction, and in the resulting mathematical relationships.

The methods to construct the open system, bond graph models are extended from the work of Redfield [1] and Karnopp [2]. This method compares in many ways with Beaman and Breedveld [3] but with perhaps a more energetically structured approach, and more emphasis on the physical interpretation of the bond graph configuration. Earlier work with bond graphs and open fluid systems is in Karnopp [4] where momentum principles develop equally valid bond graphs. More general work on Lagrangian mechanics and multibond graphs, of which the formulation of the current work is a subset, is in Breedveld and Hogan [5].

The following sections, after the nomenclature, deal with the Lagrangian bond graph development, the classical equation development, and then a comparison of the approaches and the information afforded. Simulation results from the model are shown at the end to give a flavor of the dynamics and the issues involved. Benefits of either approach all also sprinkled into the development where appropriate. The paper ends with some conclusions.

LAGRANGIAN BOND GRAPH DEVELOPMENT

A dynamic model of a water rocket is presented by first using Lagrangian bond graphs (Redfield [1]). Detail is presented here to fully describe the Lagrangian bond graph approach in the context of the rocket application.

Figure 1 is a schematic of the rocket in vertical orientation.

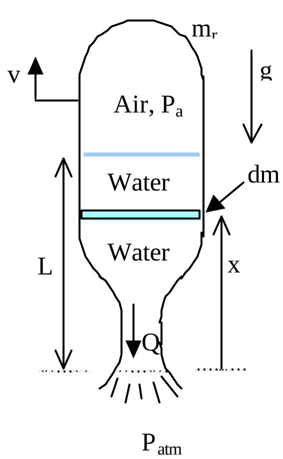

Figure 1 – Bottle rocket schematic

Air is in the top volume (accumulator) at pressure P_a and fluid is in the bottom volume of height L. The cross sectional area of the fluid volume varies over L and x is the position of a differential slice of fluid measured from the exit. The fluid of density r is assumed incompressible and its volumetric flow rate is Q, positive exiting the rocket. The rocket has a mass m_r, and a velocity v, positive upward.

Accounting for system kinetic and potential energy, adding the effects of convected kinetic energy, and then considering the energetic influence of the environment leads to a development of the bond graph model.

Accounting Kinetic Energy

We begin by modeling the kinetic energy of the system, $T_p(v,Q,V)$[1], working with the two flow variables v and Q and the displacement variable of fluid volume, V. We formulate the kinetic energy in terms of the kinetic co-energy, $T_f(v,Q,V)$[2], since it is much easier to describe due to volumetric flowrate being constant across the fluid volume. Note that $\dot{V} = -Q$ in our application. Applying the chain rule for the time derivative of the co-energy gives

$$\dot{T}_f = \frac{\partial T_f}{\partial v}\dot{v} + \frac{\partial T_f}{\partial Q}\dot{Q} + \frac{\partial T_f}{\partial V}\dot{V} = p\dot{v} + \Gamma\dot{Q} - \frac{\partial T_f}{\partial V}Q \,. \qquad (1)$$

By definition, p and G are momentum and pressure momentum respectively. $\frac{\partial T_f}{\partial V}$ is a generalized effort with units of pressure. Using the Legendre transformation for the sum of kinetic energy and co-energy (Karnopp et. al. [6]),

$$T_f + T_p = pv + \Gamma Q \,, \qquad (2)$$

with equation 1 results in the time derivative of the system kinetic energy, $\dot{T}_p$.

$$\dot{T}_p = \dot{p}v + \dot{\Gamma}Q + \frac{\partial T_f}{\partial V}Q \,. \qquad (3)$$

From equation 3 there are three power variable pairs that change kinetic energy and in a bond graph development there are three power ports that affect the kinetic energy of the system. This gives a bond graph fragment for kinetic energy storage as in Figure 2.

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[1] Subscript p denotes true kinetic energy, a function of momentum (p).

[2] Subscript f denotes kinetic co-energy, a function of flow (f).

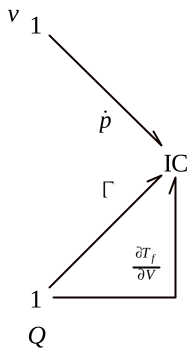

Figure 2 – Bond graph fragment representing rocket kinetic energy

An *IC*-field represents the kinetic energy because the energy is a function of momentum and displacement variables. It has three power paths affecting it. The states of the field are the two momenta, *p* and *G*, and the fluid volume *V*.

In the case of the rocket, the kinetic co-energy of the system in vertical flight includes both the rocket and fluid inertias.

$$T_f = \frac{1}{2} m_r v^2 + \frac{1}{2} \int \left(v - \frac{Q}{A(x)} \right)^2 dm \qquad (4a)$$

$A(x)$ and dm are the area and mass of the differential slice of fluid in Figure 1. With $dm = r\, A(x)dx$, the co-energy becomes

$$T_f = \frac{1}{2} m_r v^2 + \frac{r}{2} \int_0^L A(x) \left(v - \frac{Q}{A(x)} \right)^2 dx \,. \qquad (4b)$$

Taking the partial derivatives with respect to the flows give the momenta from equation 1.

$$p = \frac{\partial}{\partial v} T_f = m_t v - rLQ \qquad (5)$$

$$\Gamma = \frac{\partial}{\partial Q} T_f = -rLv + IQ$$

$m_t = m_r + rV$ is the total system mass and $I = r \int_0^L \frac{1}{A(x)} dx$ is the hydraulic inertia term for variable area sections[3].

The generalized effort of equation 1 comes from the volume dependency of the co-energy:

$$\frac{\partial T_f}{\partial V} = \frac{\partial T_f}{\partial L} \frac{\partial L}{\partial V} = \frac{?A_L}{2} \left(v - \frac{Q}{A_L} \right)^2 \cdot \frac{1}{A_L} = \frac{?}{2} \left(v - \frac{Q}{A_L} \right)^2 \qquad (6)$$

[3] Note that for constant area cross section the hydraulic inertia becomes rL/A.

The term in the parentheses is absolute water velocity at top of the water volume so $\frac{\partial T_f}{\partial V}$ is the specific kinetic co-energy at the air-fluid interface.

Accounting Potential Energy

Potential energy, V_q[4], is stored in the rocket as pressure energy in the air volume and in the fluid head of the water. Its change is represented in equation 7.

$$\dot{\mathcal{V}}_q = -P_a Q - rgLQ \qquad (7)$$

The product of accumulator pressure and volumetric flow is potential energy leaving the air volume, as is the product of fluid head pressure (rgL) and volumetric flow leaving the rocket.

The accumulator is assumed a constant mass, adiabatic process. The pressure-volume relationship is $P_a V_a^{g}$ = constant, where γ is the ratio of specific heats (γ=1.4 for air). Thus

$$P_a = P_{a-in} \left(\frac{V_{a-in}}{V_a} \right)^{?} \qquad (8)$$

with the "-in" subscript denoting initial pressure and volume.

Both effort variables in equation 7 are functions of displacement variables so *C* elements represent this stored energy as in Figure 3. Air pressure is a function of air volume, V_a and fluid head is a function of fluid height, *L*. Note that $\dot{V}_a = Q$.

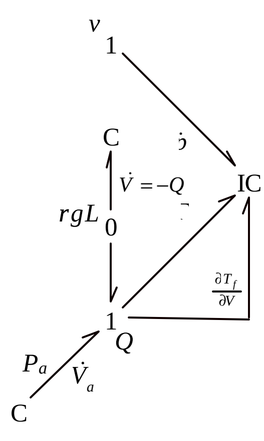

Figure 3 – Bond graph fragment of rocket incorporating potential energy

[4] True potential energy is the integral of effort (*e*) over displacement (*q*) and hence the subscript. The script "V" ($\mathcal{V}$) is distinguished from our symbol for volume (V).

Accounting Convected Kinetic Energy

The convected kinetic energy *out* of the system is the specific kinetic energy at the exit times the volumetric flow rate.

$$\dot{T}_{p-conv} = \left.\frac{\partial T_p}{\partial V}\right|_0 Q \tag{9}$$

Continuing to work with kinetic co-energy as previously, we represent the specific kinetic energy with equation 2 and its partial derivative with respect to volume evaluated at the exit plane, $|_0$.

$$\left.\frac{\partial T_p}{\partial V}\right|_0 = \left.\frac{\partial \Gamma}{\partial V}\right|_0 Q - \left.\frac{\partial T_f}{\partial V}\right|_0 + \left.\frac{\partial p}{\partial V}\right|_0 v \tag{10}$$

This leads to convected kinetic energy in terms of convected linear and pressure momentum and specific kinetic co-energy, all at the exit (equation 11).

$$\dot{T}_{p-conv} = \left(\left.\frac{\partial \Gamma}{\partial V}\right|_0 Q\right)Q + \left(\left.\frac{\partial p}{\partial V}\right|_0 Q\right)v - \left(\left.\frac{\partial T_f}{\partial V}\right|_0\right)Q \tag{11}$$

At the exit plane of the rocket, the specific pressure momentum, linear momentum and kinetic co-energy are shown in equations 12a to 12c.

$$\left.\frac{\partial G}{\partial V}\right|_0 = \frac{?}{A_0}\left(\frac{Q}{A_0} - v\right) \tag{12a}$$

$$\left.\frac{\partial p}{\partial V}\right|_0 = ?\left(v - \frac{Q}{A_0}\right) \tag{12b}$$

$$\left.\frac{\partial T_f}{\partial V}\right|_0 = \frac{?}{2}\left(v - \frac{Q}{A_0}\right)^2 \tag{12c}$$

The bond graph of Figure 3 is now modified with an *R*-field to include the convective terms of equation 12.

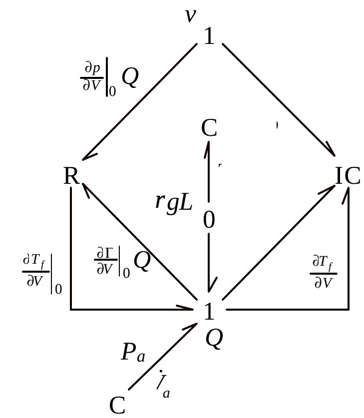

Figure 4 – Rocket bond graph including convected kinetic energy

The *R*-field is appropriate since energy loss is modeled and the effort terms of equation 11 are functions of flows. Obviously the convected pressure momentum and linear momentum are associated with Q and v respectively. The specific kinetic co-energy is a pressure-type term and is associated with Q.

Accounting Energy Exchange With The Environment

Power is lost to the environment through gravitational attraction, aerodynamic drag, and work done at the fluid exit. Quantitatively this power flow, $?_e$ (positive leaving the system), is:

$$?_e = (m_t g)v + F_d v + P_{atm} Q \tag{13}$$

$F_d = \frac{?C_d}{2} v|v|$ is the drag force (positive down) and P_{atm} is atmospheric pressure. If we add these effects to the model, the final form of the bond graph results in Figure 5[5].

System State Equations

The state equations are easily read from the bond graph. The equation representing total system linear momentum is:

$$\dot{p} = -F_d - m_t g - \left.\frac{\partial p}{\partial V}\right|_0 Q \tag{14}$$

External forces and convection change total linear momentum. The state equation for pressure momentum is:

$$\dot{G} = P_a - P_{atm} + \left.\frac{\partial T_f}{\partial V}\right|_0 - \frac{\partial T_f}{\partial V} - \left.\frac{\partial G}{\partial V}\right|_0 Q + ?gL \tag{15}$$

[5] If you look closely, the bond graph even resembles the rocket.

Pressure momentum, which is positive downward, is changed first by the pressure differential across the fluid. The next two terms (partial derivatives) represent the difference between the specific co-energy at the exit and that at the air-water interface. Disregarding other effects, if the specific co-energy is greater at the exit than the interface then G (which is the change in T_f with respect to Q) is increasing with time. The third partial is the convection of pressure momentum out of the rocket and the last term is the fluid head effect that acts as an effective pressure.

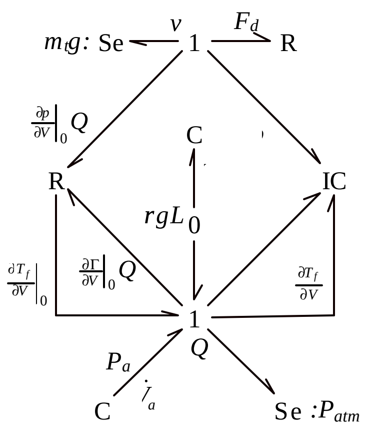

Figure 5 – Completed bond graph of water rocket.

Alternative Formulation

The proceeding formulation was based mostly on energetic considerations. It is generally known that with bond graphs, if you get the "kinematics" correct, the "kinetics" are "free" since power is conserved. After the original kinetic energy accounting, we could have started with the linear momentum statement of equation 14 (which is easily written directly) and added the system power balance of equation 16.

$$-(F_d - m_t\, g)v - P_{atm}Q - \left.\frac{\partial T_p}{\partial V}\right|_0 Q = \dot{T}_p + \dot{V}_q \tag{16}$$

Equation 16 has the power interactions with the environment, the convected energy, and the rate of change of system kinetic and potential energy. Equations 14 and 16, along with previous relationships for $\left.\frac{\partial p}{\partial V}\right|_0$, $\left.\frac{\partial T_p}{\partial V}\right|_0$, $\dot{T}_p$, and $\dot{V}_q$ result (after a bit of manipulation) in the pressure momentum result of equation 15 and thus the same system bond graph.

Equations Of A More Common Form

It is interesting to see that if we substitute derivatives of equation 5 into our state equations 14 and 15, we get the more standard equations:

$$m_t\,\dot{v} - ?L\dot{Q} = -m_t\, g + ?Q^2\left(\frac{1}{A_0} - \frac{1}{A_L}\right) - F_d \tag{17a}$$

$$I\dot{Q} - ?L\dot{v} = \frac{?Q^2}{2}\left(\frac{1}{{A_L}^2} - \frac{1}{{A_0}^2}\right) + P_a - P_{atm} + ?gL \tag{17b}$$

These can be found, for instance, in Karnopp [1972] for one-dimensional motion.

CLASSICAL MODELING APPROACH

A more classical approach to the equation development is examining a differential fluid element and summing forces. An element of fluid is shown at position x with cross-sectional area A. A free body diagram of the differential element is shown in Figure 6 where x is the measure to the center of the element and A is the cross sectional area, which can change with x. Pressure and fluid head effects, are considered.

Summing forces on the element gives

$$\left[P + \frac{\partial P}{\partial x}(-\Delta x/2)\right]\left[A + \frac{\partial A}{\partial x}(-\Delta x/2)\right] -$$
$$\left[P + \frac{\partial P}{\partial x}(\Delta x/2)\right]\left[A + \frac{\partial A}{\partial x}(\Delta x/2)\right] - rAg\Delta x +$$
$$P\left[A + \frac{\partial A}{\partial x}(\Delta x/2) - \left(A + \frac{\partial A}{\partial x}(-\Delta x/2)\right)\right] = rA\Delta x\frac{d}{dt}\left(v - \frac{Q}{A}\right)$$

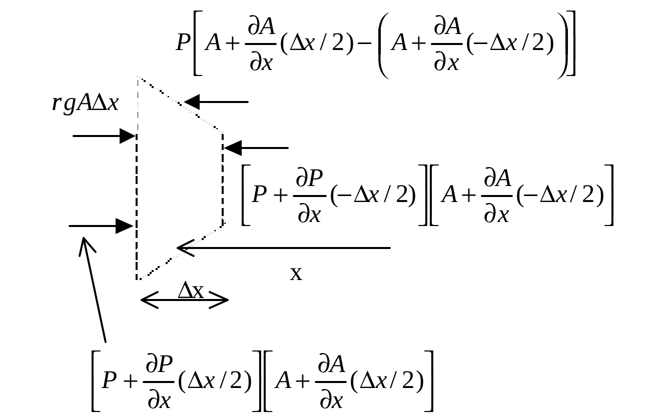

Figure 6 - Differential element in classical approach

Simplification of this equation and taking the right side derivative gives:

$$-\,?g - \frac{dP}{dx} = ?\left(\dot{v} - \frac{\dot{Q}}{A} + \frac{Q^2}{A}\frac{d(1/A)}{dx}\right) \tag{18}$$

And this integrates to equation 17b.

Equation 17b is a second order differential equation in two unknowns (v and Q) so another relationship is necessary to make the system complete. We do a momentum balance on the entire system and end up with equation 19.

$$-F_d - m_t g - r\left(v - \frac{Q}{A_o}\right)Q = \frac{d}{dt}\left(m_t v - rQL\right) \qquad (19)$$

Equation 19 shows two force terms, a momentum convection term, and a time rate of change of system momentum. Taking the derivative in equation 19 and arranging gives equation 17a. We now have a coupled pair of second order equations that is ready for computer solution.

A COMPARISON OF APPROACHES

To study the benefits of the two formulation approaches, we can compare information in the free-body diagram to that in the bond graph and we can compare the equation information in the bond graph to those of equation 17. What do these tell us about the water rocket system?

The FBD probably gives more insight on the micro-level. Forces that act on a differential slice are not apparent in the bond graph approach although the interpretation of the FBD just gives forces on the differential element due to pressure and gravity. Further, the resulting classical equations are a little hard to interpret. Equations 17a and b are coupled in *v* and *Q*, which make interpretation more difficult. Equation 17a sums force-like terms to give changes in velocity and flow rate. The first term on the left-hand side of 17a is the total mass times absolute acceleration, the first and last terms on the right are gravitational and drag forces, and the term in parentheses is a difference in specific, relative momenta. Equation 17b is likewise. There are pressure terms and something we would call dynamic pressure although this terms deal with relative velocities (the first term on the right side). The two parts of 17 could be combined to give a single second order equation but the interpretation of these terms might be even tougher.

The bond graph itself, however, gives specific information about linear momentum, pressure momentum, and energy accounting. For example in Figure 5, the *1* junction associated with volumetric flow, *Q*, shows that (absolute) pressure momentum is changed by a static pressure difference, fluid head, pressure momentum convection, and the difference between specific kinetic co-energies at the two ends of the fluid volume. What do these specific energies physically mean? They are pressure-like terms that account for a difference in cross-sectional areas at the air-water interface and the exit. If there are no other affects, more specific kinetic co-energy at the exit than the interface means an increase in pressure momentum.

The *1* junction associated with the linear velocity shows the total momentum rate equal to the net external force minus the convected momentum.

The bond graph shows explicitly what affects the kinetic energy of the whole system (the *IC*-field), and what terms combine to represent convected energy (the *R*-field). The bonds on the IC field indicate that the kinetic energy is changed by mechanical power, hydraulic power and the change in volume of fluid. The three ports that sum to the convected energy include convected linear and pressure momentum, and specific kinetic co-energy at the exit.

It must be said that the relationships from the bond graph are possible to come by through classical methods, but the methods themselves would not thrust them upon you. The Lagrangian bond graph approach gives these in the normal course of model formulation.

SOME SIMULATION RESULTS

Simulations were run varying the initial air pressure and initial fill ratio of the rocket. The following results are for an initial pressure of approximately 5.5 atmospheres. It was found that an initial fluid percentage of about 42% provided the highest rocket altitude for this initial pressure. Optimal initial fluid percentage varies with launch pressure. The following results are for a fill of 40% and the simulations are run until the fluid is totally emptied from the rocket.

The first graph is the total momentum of the system, rocket body and water (Figure 7). This momentum starts slowly for the first milliseconds and then increases to a maximum near the time the fluid empties. Examining the bond graph and seeing that the main component to momentum change is convection can explain the delay in total momentum. Gravity and drag are not big players here as seen in Figure 8. This shows the convection (Cnv), gravitational (Grv), and drag (Drg) contributions to momentum change.

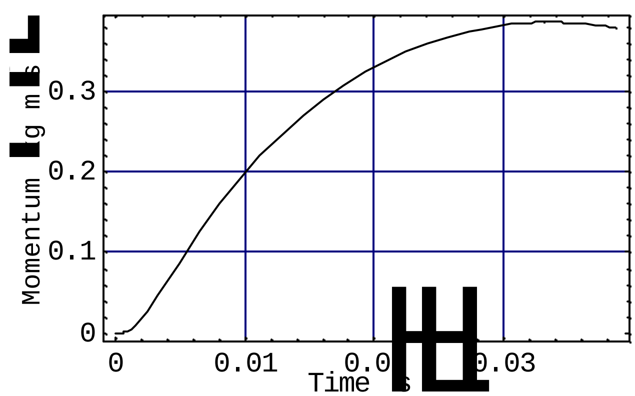

Figure 7 – Total system momentum

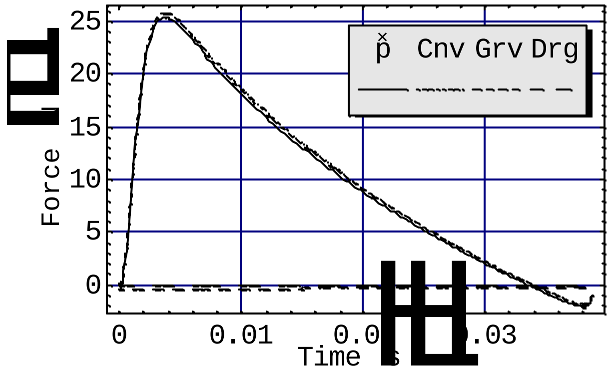

Figure 8 – Contributions to momentum change

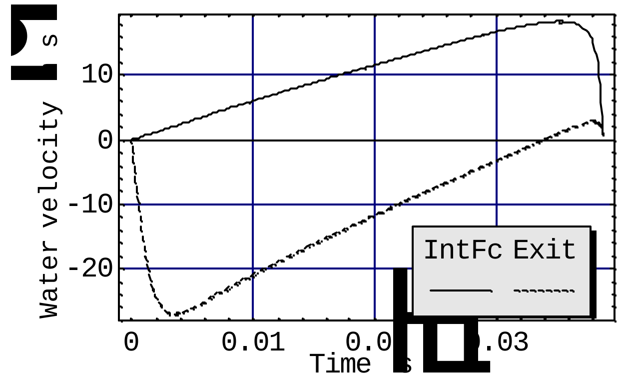

Figure 10 – Absolute velocities at interface and exit

The pressure momentum is shown next in Figure 9 with the positive direction downward. *G* starts out quickly positive due to the initial volumetric flow rate (*Q*), ends up negative when the rocket picks up sufficient speed, and ends necessarily at zero when the rocket has no more water.

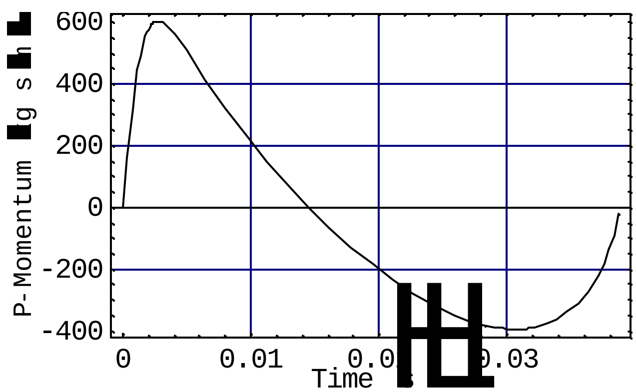

Figure 9 – Pressure momentum

It is a bit difficult to think of the total pressure momentum because it is integrated across the fluid volume and the absolute velocity at any point of the fluid is a combination of rocket velocity (*v*) and *Q*. The fluid at the interface (IntFc) is moving upward absolutely while the fluid at the exit is moving downward for most of the time (Figure 10).

Fluid at the interface and exit are necessarily equal when the fluid empties but why these velocities are zero is not clear.

The contributions to pressure momentum can be seen in the pressure terms of Figure 11. There is net pressure (Pr), convection of pressure momentum (Cnv), and specific kinetic co-energy difference (STf). The pressure initiates *G* but convection and specific kinetic co-energies quickly contribute as the flow rate increases. Pressure momentum rate goes negative after only a few milliseconds due mostly to convection. Pressure momentum convection behaves much as the linear momentum convection did (Figure 8). The specific co-energies and convection end at zero as they should.

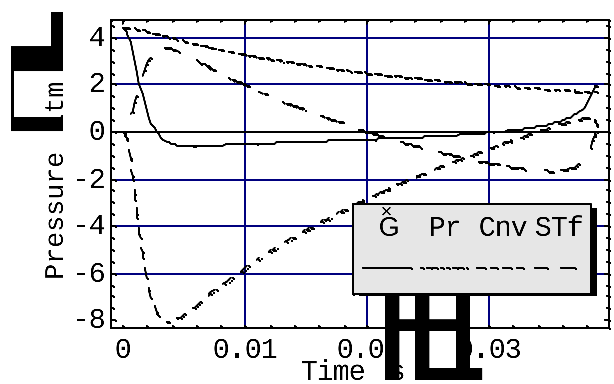

Figure 11 – Contributions to pressure momentum

The net results of these dynamics are the rocket velocity of Figure 12 and the volumetric flow rate of Figure 13. (Note that mass flow is just the product of *Q* and *r*.) After the first milliseconds, the rocket appears to have a nearly constant acceleration. The reason for this is not clear. *Q* jumps up initially and slowly decreases till the water is depleted.

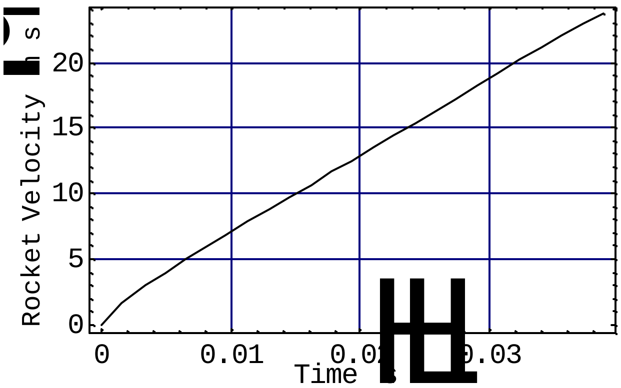

Figure 12 – Rocket velocity

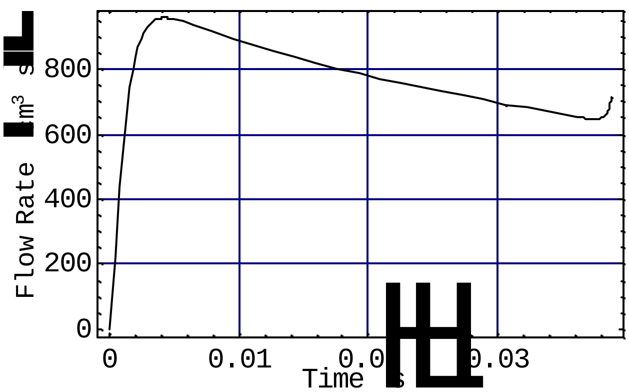

Figure 13 – Water volumetric flow rate

Figure 14 shows the main energy contributions to the dynamics of the water rocket. KE_t is the total kinetic energy, that of the rocket body plus that of the rocket fluid. KE_b is the kinetic energy of the rocket body only. Both energies start at zero and are equivalent at the end when the water is gone. The body kinetic energy is parabolic which is consistent with a mostly linear increase in rocket velocity.

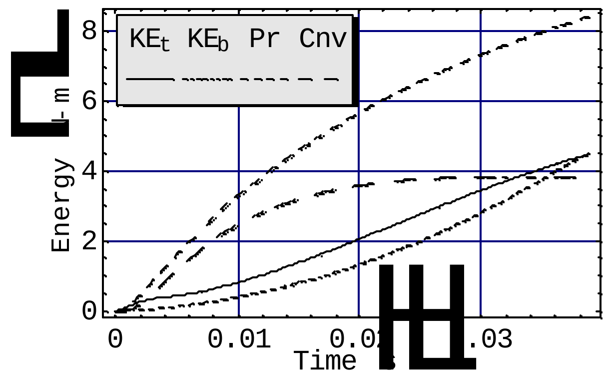

Figure 14 – Energetic contributions to rocket dynamics

The main players in this energy accounting are the energy delivered by the accumulator due to a pressure expansion (Pr) and the energy convected out of the rocket (Cnv). The total kinetic energy is nearly equal to the pressure energy minus the convected energy. Other players are the energies due to drag (Drg), fluid head (Hd), and gravitational potential (Grv). These are shown to be negligible in Figure 15 when compared with Figure 14 magnitudes.

The pressure energy is approximately a decreasing exponential: as the pressure drops, its delivery of energy flattens out. The convected energy increases initially and then flattens to nearly a zero rate of change for the last quarter of the simulation. Since the convected energy is a function of the absolute fluid velocity and the volumetric flow rate, Figures 10 and 13 are reexamined. Although the flow rate remains high at the end, the absolute fluid velocity of the exit is low and crosses zero. Even though fluid is expelled, its energy is very low since its absolute velocity is small.

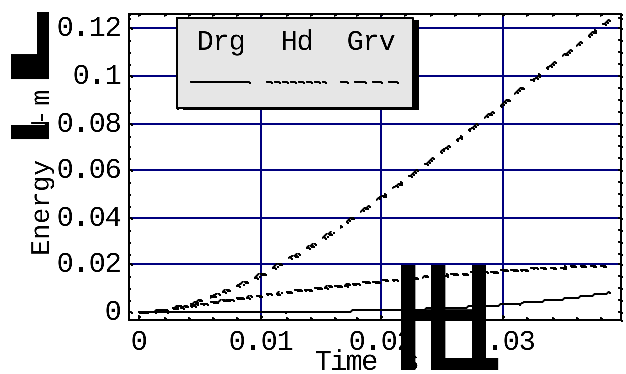

Figure 14 – Minor energetic contributions

CONCLUSIONS

This paper focused on the development of open system models using Lagrangian bond graphs and the insight gained in both the model formulation and the examination of simulation results. A classical model was similarly formed and was shown to yield an equivalent set of equations of motion. The bond graph model development was certainly more involved but this is exactly what afforded insight into relationships in the dynamics. The classical development was more concise, but the resulting equations were not as helpful.

Of course once v and Q are solved for classically, the other variables of interest could be constructed as output equations but the accounting that shows the contributors to each important variable would probably be obscured in the math.

A benefit of the bond graph approach not discussed is the ease of changing modeling assumptions. If fluid frictional forces are added, a resistive element associated with the 1_Q junction is appended. Other effects are easily deleted such as drag or fluid head and the result on key variables is immediately seen in the bond graph.

These conclusions can also be scaled with system complexity since larger systems are mostly handled by reticulating models into sub-systems and components.

A net result of this work not only shows how to systematically and logically construct Lagrangian bond graphs from purely energetic considerations, it shows the many relationships that bond graphs generate for physical variables. Even though the author has yet to determine what exactly contributes to the rockets maximum altitude, he is convinced that answer is available with further pondering over both the bond graph model and the simulation results.

ACKNOWLEDGMENTS

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REFERENCES

[1] Redfield, Robin C., 1995 “A Bond Graph Representation of Lagrange's Equations,” Proceedings *of the ASME Dynamic Systems and Control Division*, ASME International Mechanical Engineering Congress and Exposition, DCS-Vol. 57-2, pp. 431-438.

[2] Karnopp, D., 1992, “An Approach to Derivative Causality in Bond Graph Models of Mechanical Systems*,” Journal of the Franklin Institute*, Vol 329, No. 1, pp. 65-75.

[3] Beaman, J. and Breedveld, P., 1988, “Physical Modeling with Eulerian Frames and Bond Graphs,” *Journal of Dynamic Systems, Measurement, and Control*, Vol 110, June, pp. 222-229.

[4] Karnopp, D., 1972, “Bond Graph Models for Fluid Dynamic Systems,” *Journal of Dynamic Systems, Measurement, and Control*, September 1972, pp. 222-229.

[5] Breedveld, P. and Hogan, N., 1994, “Multibond-Graph Representation of Lagrangian Mechanics: The Elimination of the Euler Junction Structure,” *Proceedings IMACS 1. MathMod Vienna*, Technical University, Vienna, Austria, February, pp. 24-28.

[6] Karnopp, D., Margolis, D., and Rosenberg, R., 1990, *System Dynamics; A Unified Approach*, Wiley InterScience, New York.